A 2D model of Causal Set Quantum Gravity: The emergence of the continuum.

Graham Brightwell[1], Joe Henson[2] and Sumati Surya[3]
[1]London School of Economics, London, UK,
[2]Perimeter Insitute, Waterloo, Canada
& University of Utrecht, Utrecht, Netherlands,
[3]Raman Research Institute, Bangalore, India

September 17, 2008

Abstract

Non-perturbative theories of quantum gravity inevitably include configurations that fail to resemble physically reasonable spacetimes at large scales. Often, these configurations are entropically dominant and pose an obstacle to obtaining the desired classical limit. We examine this "entropy problem" in a model of causal set quantum gravity corresponding to a discretisation of 2D spacetimes. Using results from the theory of partial orders we show that, in the large volume or continuum limit, its partition function is dominated by causal sets which approximate to a region of 2D Minkowski space. This model of causal set quantum gravity thus overcomes the entropy problem and predicts the emergence of a physically reasonable geometry.

In approaches to quantum gravity where the continuum is replaced by a more primitive entity, manifoldlikeness is typically a feature of only a small proportion of the configurations. In order to obtain the correct continuum limit, this small set of configurations needs to be dynamically favoured over the often overwhelming entropic contribution from non-manifoldlike configurations. It has been argued that

some form of this "entropy problem" is of critical importance in dynamical triangulations, graph-based approaches and in causal set quantum gravity(CSQG) [1]. This present work shows how the problem is overcome in a simplified 2D model of CSQG.

In CSQG, continuum spacetime arises as an approximation to a fundamentally discrete structure, the causal set. Here, order and number correspond to the continuum notions of causal order and spacetime volume. Despite being discrete, local Lorentz invariance in the continuum approximation is restored by using a random lattice [2]. These basic features of the theory led to an early prediction for the cosmological constant, confirmed several years later by observation [3]. The construction of a model of CSQG with physically realistic predictions is therefore of considerable interest.

The fundamental entity that replaces spacetime in CSQG is a causal set, or causet, $(C, \prec)$, which is a locally finite partially ordered set. Namely, for any $x, y, z \in C$ (i) $x \not\prec x$ (irreflexive)[1], (ii) $x \prec y$, $y \prec z \Rightarrow x \prec z$ (transitive) and (iii) Cardinality(Future$(x) \cap$ Past(y)) is finite (locally finite), where Future$(x) = \{z|x \prec z\}$, and Past$(x) = \{z|z \prec x\}$. Local finiteness means that discreteness is taken to be fundamental, and not simply a tool for regularisation. The continuum (M, g) arises as an approximation of a causet C if there exists a "faithful embedding" $\Phi : C \rightarrow (M, g)$ at density V_c^{-1}, where V_c is the discreteness scale. This means that (a) the distribution of $\Phi(C) \subset M$ is indistinguishable from that obtained via a Poisson sprinkling into (M, g), i.e., a random discrete set S such that, for any region R of volume V, the number of points of S in R is a Poisson random variable with mean V/V_c, and (b) the order relation $\prec$ in C is in 1-1 correspondence with the induced causal order on $\Phi(C)$, i.e., $x \prec y \Leftrightarrow \Phi(x)$ is to the causal past of $\Phi(y)$ [4, 5].

While a quantum version of a classical sequential growth dynamics for causets may eventually provide a more natural framework for quantisation [6], it is useful to consider the standard path-integral paradigm. As in other discrete approaches [7], the path integral is replaced by a sum, which in CSQG is over causets, with an appropriate "causet action" providing the measure. The Regge action is an obvious choice for discrete theories of quantum gravity based on triangulations of spacetimes

[1]This can be replaced by the condition that $x \prec y$, $y \prec x \Rightarrow x = y$. For both choices one avoids causal "loops".

[8]. However, in CSQG, because of the intrinsic non-locality of causets, an action defined as a sum of strictly local quantities is likely to fail. The construction of a causet action is deeply intertwined with the question of locality and the associated problems in constructing a consistent quantum dynamics [9, 10]. While prescriptions for a localised D'alembertian [10] may eventually lead to an approximately local action for causets, it is a worthwhile exercise to sidestep this question by considering simplified models.

One possible approach is to make a precise definition of the class of "manifold-like" causets, and restrict the history space to this class. Such causets have a natural corresponding continuum action which can be used to define the partition function. While manifoldlikeness is a trivial prediction of such a model, it may nevertheless display features that yield interesting insights into CSQG. Without further restrictions, however, such a model is not obviously tractable. One such restriction is by spacetime dimension, the simplest choice being dimension 2.

The specific model of 2D CSQG we present here is constructed via a restriction to the class of so-called 2D orders. This class contains not only all causets that approximate to conformally flat 2D spacetime intervals, but also some that are non-manifoldlike. Moreover, all causets in this class share a certain topological triviality. This allows us to meaningfully address the entropy problem and the question of manifoldlikeness within the model. We find that the entropy problem is tamed in an unexpected way, and that it is possible to characterise its physical consequences with results that may be surprising.

Such a model can be regarded as a restriction of the full theory of CSQG, analogous to mini and midi-superspace models in canonical approaches to quantum gravity – the hope would be to gain insights into the full theory by understanding details of the simplified model. Indeed, our model is more fully dynamical than such reduced models, because rather than freezing local degrees of freedom, one is simply restricting to a class of causal sets that are naturally associated with discretisations of 2D spacetime, with a fixed topology. Causal set theory does not in principle assume a fixed spacetime dimension, and hence our 2D model is indeed a restriction of the full theory. However, this simply brings it on par with the starting point of other routes to quantisation which must assume a fixed spacetime dimension. While a restriction to topology is routinely adopted in other approaches to quantum gravity, the hope is that our model can ultimately be generalised to include a sum over

all 2D topologies.

Although our work does not lead directly to a 4D theory, it is an example of how the continuum can be recovered from a quantum causet model and hence may prompt more optimism on the general approach presented above. Moreover, it is an explicit demonstration that the causet approach is rich enough to allow formulations with physically sensible outcomes, without the addition of extra variables [11].

We consider a causet "discretisation" of the set of 2D conformally flat spacetime intervals (I, g). Using a fiducial flat metric, $\eta_{ab}dx^a dx^b = -dudv$ in light cone coordinates (u, v), these geometries are represented by diffeomorphism classes of the metrics

$$g_{ab}dx^a dx^b = -\Omega^2(u, v)\, dudv, \tag{1}$$

with $\Omega^2(u, v)$ the conformal factor. Quantisation of this set of spacetimes on I can be thought of as a Lorentzian analog of Euclidean 2D quantum gravity on a disc [12]. As a topological space, I is simply homeomorphic to a disc, with the boundary condition that there exists an interval I_0 of ${}^2\mathbb{M}$ and a bijection $\Psi : \partial I \to \partial I_0$. In 2D the conformal factor encodes all geometric degrees of freedom so that all Lorentzian metrics on this manifold have the form (1).

We will also adopt the unimodular modification of gravity, in which spacetime volume plays the role of a time parameter [13]. Fixing the time coordinate is thus given a covariant meaning, corresponding to the volume constraint

$$\mathcal{V} = \int \Omega(u, v)\ dudv = \text{constant}. \tag{2}$$

This constraint places restrictions on the map Ψ; starting with an "initial" event $p_0 = (u_0, v_0)$ in the fiducial metric η_{ab}, the "final" event $p_f = (u_f, v_f)$ of the interval $I_0 = [(u_f, v_f), (u_0, v_0)]$ in $(\mathbb{R}^2, \eta_{ab})$ is determined (upto boosts) by the condition $\int_{u_0}^{u_f}\int_{v_0}^{v_f} \Omega(u, v)\, dudv = \mathcal{V}$.

The Einstein action on an interval includes a term on the null boundary. In order to simplify the action, we take the interval I to be enlarged ever so slightly, to a region $I' \supset I$ with spacelike boundary components. Because of the nature of the manifold approximation, a causet discretisation is insensitive to modifications of the boundary on scales much smaller than the discretisation scale, i.e. if the volume of the region $I' - I$ is $\ll V_c$. Since the boundary of I' is piecewise spacelike, the

Einstein action takes the form

$$S = \frac{1}{16\pi G}\int_{I'} RdV - \frac{1}{8\pi G}\int_{\partial I'} kdS - \sum_j \frac{1}{8\pi G}\theta_j - \frac{1}{8\pi G}\Lambda V_{I'}, \tag{3}$$

where θ_j are the four boost parameters corresponding to the four "joints" in $\partial I'$ [14, 15]. The Lorentzian Gauss Bonnet theorem [16, 17] then simplifies the action to

$$S = \frac{1}{16\pi G}(2\pi i - 2\Lambda V_{I'}), \tag{4}$$

which is a constant over the entire class of spacetimes under consideration.

The first step in constructing the causet discretisation of this continuum theory is to characterise the class of causets of finite cardinality which embed faithfully into conformally flat 2D intervals. While such a characterisation appears daunting in general, for our model these causets lie in the set of "2D orders", a well-studied class of partially ordered sets.

To define this class, some nomenclature is necessary. Consider a set of elements $S = \{e_1, ...e_n\}$ and a partial order $\prec$ on this set. A causet X on the underlying set S is a linear order if and only if, for all i, j, $e_i \prec e_j$ or $e_j \prec e_i$ in X. We use the notation $Q = (e_{\pi(1)}, e_{\pi(2)} \ldots e_{\pi(i)}, \ldots e_{\pi(n)})$ to denote a linear order on S, where π is a permutation on n elements, so that $e_{\pi(i)} \prec e_{\pi(i+1)}$ for all i. A linearly ordered subset of a causet is known as a chain. Similarly, a totally unordered causet is one such that $e_i \not\prec e_j$ for all i, j, and a totally unordered subset is known as an antichain. For causets $Q_1, Q_2, \ldots, Q_k$ on the same set S, the intersection $P = \bigcap_{i=1}^k Q_i$ is defined by setting $e_i \prec e_j$ in P if and only if $e_i \prec e_j$ in all of the Q_i. For example, if $Q_1 = (e_1, e_2)$ and $Q_2 = (e_2, e_1)$, then since $e_1 \prec e_2$ in Q_1 and $e_2 \prec e_1$ in Q_2, they are unrelated the intersection $Q_1 \cap Q_2$ which is therefore a two element antichain. The "dimension" of a causet P is the minimum k such that P can be written as the intersection of k linear orders. Our main interest is in "2-dimensional" or 2D orders: ones that can be written as the intersection of two linear orders, but are not themselves linear orders[2] (see Fig 1).

In the rest of this paper we will say that a causet C "corresponds" to a given spacetime (M, g) if there exists a faithful embedding $\Phi : C \to (M, g)$, defined

[2]Note that causet dimension in this context is not apriori related to the spacetime dimension.

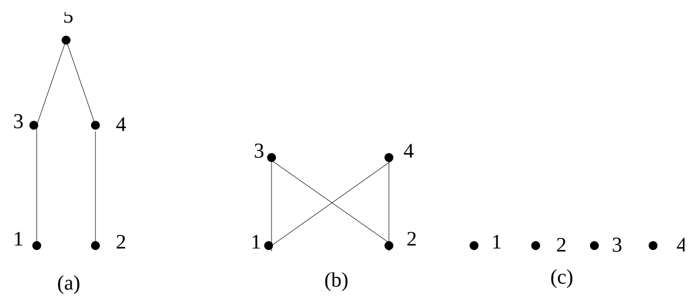

Figure 1: Examples of labelled 2D orders, obtained from the intersections of the following linear orders: (a) $L = (e_1, e_3, e_2, e_4, e_5)$ and $M = (e_2, e_4, e_1, e_3, e_5)$ (b) $L = (e_1, e_2, e_3, e_4)$ and $M = (e_2, e_1, e_4, e_3)$ and (c) $L = (e_1, e_2, e_3, e_4)$ and $M = (e_4, e_3, e_2, e_1)$.

precisely by Bombelli as follows [5]. Let $\Phi : C \to M$ be an embedding of a causet C of cardinality V/V_c into a spacetime of finite volume V. Consider sampling intervals of volume $V_c < V_0 < V$. Then

$$P_{V_0}(n) \equiv \frac{1}{n!} e^{-\frac{V_0}{V_c}} \left(\frac{V_0}{V_c} \right)^n \tag{5}$$

is the probability of finding $n < N$ elements of $\Phi(C)$ in a region of volume V_0 for a Poisson embedding. Define the indicator function $F_n = \int \chi_n(I) dI / \int dI$, for the embedded causet $\Phi(C)$, where $\chi_n(I) = 1$, or 0 depending on whether the interval I (of volume V_0) has n points in it or not, and the integral is over all possible intervals I of volume V_0 in (M, g). Then, if $|F_n - P_{V_0}| < \delta$, Φ will be said to be a δ-faithful embedding with respect to V_0. We will henceforth use the phrase "faithfully embeddable" to imply in the (δ, V_0) sense. Specifically, we will require that $V_c \ll V_0 \ll V$ and $0 < \delta \ll 1$. For suitable choices of δ and V_0, a causet generated by a Poisson sprinkling into M with density V_c^{-1} will be, with high probability, faithfully embedded in M. On the other hand, regular discrete lattices tend not to be faithfully embedded: the regular structure leaves large intervals void of points.

To see that 2D orders are appropriate for our purposes, consider a conformally flat 2D spacetime (M, g). The causal order $\preceq$ between events p and q in such a spacetime can be encoded in the statement:

$$(u_1, v_1) \preceq (u_2, v_2) \quad \Leftrightarrow \quad u_1 \leq u_2 \quad \text{and} \quad v_1 \leq v_2, \tag{6}$$

where $(u_{1,2}, v_{1,2})$ are light cone coordinates of p and q, respectively. For conformally flat spacetimes any choice of light cone coordinates is such that the ordering on each co-ordinate u or v is a linear order. This means exactly that a finite causet can be embedded in (M, g) if and only if it is the intersection of the two co-ordinate linear orders, i.e., if and only if its "dimension" is at most 2 [18].

Although every 2D order can be embedded into a conformally flat 2D spacetime, not all of them can be *faithfully* embedded. For example, the intersection of the linear orders $L = (e_1, e_2, e_3, e_4, \ldots, e_N)$ and $M = (e_2, e_1, e_3, e_4, \ldots, e_N)$ has an antichain $\{e_1, e_2\}$, while all other e_i are to the future of both e_1 and e_2, and linearly ordered. Thus $L \cap M$ is *almost* a chain, except for the past-most two elements $\{e_1, e_2\}$, and so clearly does not faithfully embed into a 2D spacetime, at least for N sufficiently large. Thus, in this sense, not every 2D order corresponds to a 2D spacetime.

But can a 2D order faithfully embed into a spacetime of a different topology than the interval? Consider, for example, a flat 2D interval (I, η) with a large region R cut out of it (see Fig 2 (a)). $(I - R, \eta)$ is not a causally convex subset of (I, η) and hence its intrinsic causal order differs from that of (I, η). In particular, it contains pairs of events $p = (u_1, v_1), q = (u_2, v_2)$ such that $p \prec q$ in (I, η), but $p \not\prec q$ in $(I - R, \eta)$, so that $u_1 < u_2$, $v_1 < v_2$ does not imply that $(u_1, v_1) \prec (u_2, v_2)$. This means that a causal set C that faithfully embeds, via some Φ, into $(I - R, \eta)$ cannot be realised as an intersection of the lightcone coordinates of $\Phi(C)$, for R large. However, it is always possible to make appropriate changes in the embedding density in order to construct an embedding $E : C \to (I, \eta)$: E will not be faithful, but C can be realised as the intersection of the lightcone coordinates of $E(C)$ and is hence a 2D order. It would thus appear that the class of 2D orders includes faithful embeddings into intervals with regions cut out of it, i.e., topologies different from the disc. On the other hand, it is always possible to choose an interval spacetime (I, g), with a $g = \Omega(u, v)^2\eta$ which "compensates" for the varying embedding density of $E(C)$ in (I, η), so that $\Phi : C \to (I, g)$ is a faithful embedding (see Fig 2 (b)) Relevant to our model, is the resulting statement that a 2D order which embeds into an interval spacetime with holes can equivalently be obtained as a discretisation of a conformally flat interval spacetime.

The class of continuum manifolds of typical interest in 2D quantum gravity are ones with spacelike boundaries representing an initial and a final time. From this perspective, the extension of I to I' seems reasonable, since I' has both initial and

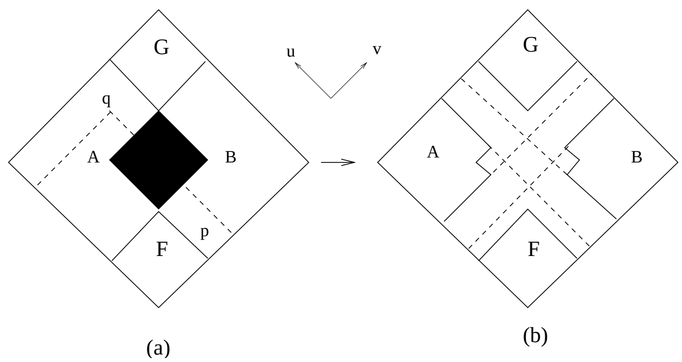

Figure 2: (a) If $\Phi : C \rightarrow (I-R, \eta)$ is faithful, $\Phi(C)$ uniformly populates the regions A, B, F, G. (b) A suitable change in the density of the embedding pushes the elements of C in regions A, B, F, G of (a) into a portion of the spacetime (I, η) without changing the order-causality correspondence. By choosing a conformal factor $\Omega(u, v)$ which is approximately one in the regions A, B, F, G and vanishingly small elsewhere, $\Phi(C)$ can be equivalently thought of as a faithful embeddedding into $(I, \Omega^2(u, v)\eta)$.

final spacelike boundaries. Now, except spacetimes on the interval topology, all 2D spacetimes of finite volume which satisfy this boundary requirement have non-contractible loops and non-vanishing first Betti numbers β_1. β_1 is also non-vanishing for any spatial slice in these spacetimes. In [19] it was shown that causal sets C that faithfully embed into a globally hyperbolic region of a spacetime contain sufficient structure to reproduce the spatial continuum homology with high probability. The construction in [19] uses the idea of a thickened antichain from which a nerve simplicial complex is constructed. In particular, it can be shown that if $\beta_1 \neq 0$ for this nerve simplex, it implies the existence of a "crown" sub-poset in C. A crown poset is defined as follows. Let C have cardinality, $2m$, $m > 2$, and let $A_1 = (e_1, e_2, \ldots e_m)$, $A_2 = (e'_1, e'_2, \ldots e'_m)$ be two non-intersecting antichains in C, whose elements are related to each other by $e_i \prec e'_i, e'_{i+1}$ and $e'_i \succ e_{i-1}, e_i$, where we are treating indices modulo m (see Fig 3 for an example). We note that:

Claim 1 *A 2D order cannot contain a crown poset with $m > 2$.*

Proof Suppose the crown C_m is the intersection of linear orders L and M. Let

e'_i be the lowest primed vertex in L. Then e_i and e_{i-1} both appear below all the primed vertices in L. Now suppose wlog that e_i appears above e_{i-1} in M. Then e'_{i+1} appears above e_i in M, and hence is above e_{i-1} in M as well as in L. As e'_{i+1} is not above e_{i-1} in the crown C_m, this contradicts the assertion that $L \cap M = C_m$. $\square$

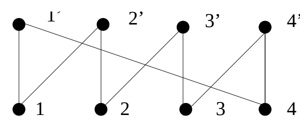

Figure 3: An eight element crown poset, constructed from $A_1 = (e_1, e_2, e_3, e_4)$ and $A_2 = (e'_1, e'_2, e'_3, e'_4)$.

To see this result within the wider context of the theory of poset dimension, the reader should consult [18]. What one would like to deduce from this is that 2D orders are exclusively associated with the interval topology. Of course, if the scale of the topology is of order the discretisation scale, then this is no longer the case. However, such continuum structure is considered irrelevant from the causet perspective, and hence discretisation of such spacetimes is not pertinent. Thus, within these limitations we may conclude that if a 2D order approximates to a 2 dimensional spacetime, then the latter belongs to the class of conformally flat interval spts. It is in this sense that the class of 2D orders distinguishes the topology of the interval from all the others relevant to 2D quantum gravity. The set of *all* 2D orders is thus a meaningful causet discretisation of the class of 2D interval spacetimes.

We are now in a position to write down the causet partition function. For 2D orders that do have a continuum approximation, our discretisation gives us a uniform measure coming from the continuum action (3). Moreover, since the set of all 2D orders includes all causets corresponding to 2D intervals, but none corresponding to other 2D spacetimes, it is natural to extend this uniform measure on manifoldlike 2D orders to all 2D orders. The partition function for our model is thus the unweighted sum over the set Ω_{2D} of unlabelled 2D orders

$$\widetilde{Z} = (\text{phase}) \times \sum_{\Omega_{2D}} 1. \tag{7}$$

The appearance of a uniform weight in the partition function comes from the triv-

iality of the continuum theory, and at first glance suggests that any semi-classical regime will be impossible in the model. In path integral quantum mechanics, for example, the set of all paths is dominated by non-classical, non-differentiable paths. The inclusion of the non-trivial weight $\exp(i\frac{S(\gamma)}{\hbar})$ is crucial in obtaining the correct classical limit.

Indeed, as shown in [20], a uniform measure over the set of all N element causets, not just those which are 2D orders, is completely dominated in the large N limit by the Kleitman-Rothschild three-level (or three "moments-of-time") causets, which are most non-manifoldlike. It is hoped that a suitable action for causets would repair this entropic problem and yield the correct continuum approximation or classical limit. In our model, however, since the action is trivial, the partition function is determined solely by entropic effects. Nevertheless, because our measure vanishes on all N element causets which are not 2D orders, a meaningful continuum approximation does indeed emerge from this theory as we will discuss below.

As labels are the discrete analogues of coordinates they are considered unphysical in causet theory. Our interest therefore lies with isomorphism classes of labelled 2D orders, i.e., with unlabelled 2D orders. The random variable $U(N)$ on the isomorphism classes of labelled 2D orders each taken with equal probability therefore matches the normalised partition function for our model (7). We will also be interested in so-called labelled *random 2D orders* $P(N) \equiv L \cap M$ which are random variables defined by choosing L and M randomly and independently from the $N!$ linear orderings of $\{e_1, ..., e_N\}$. The study of random k-dimensional orders was initiated in the 1980s by Winkler [21]. The case $k = 2$ has been of particular interest, because of its connection to random permutations, and to Young tableaux. The typical structure of a random 2-dimensional order is now reasonably well understood.

From the perspective of CSQG, this model of random orders plays a crucial role. Indeed, a random order from this model can equivalently be generated by taking a sequence of N independent random points in a fixed interval I of 2D Minkowski spacetime, according to the volume measure [21]. This in turn is equivalent to the Poisson process (or sprinkling) in the interval, conditioned on the number of points being N [22]. The equivalence of the two models can be seen as follows.

Let I be the interval of 2D Minkowski spacetime between two points a and b, with lightcone coordinates (u_a, v_a) and (u_b, v_b) respectively. Thus I is the rectangle consisting of all points with u-coordinate in $[u_a, u_b]$ and v-coordinate in $[v_a, v_b]$.

Now let C be the causet, with elements $\{e_1, \ldots, e_N\}$, obtained by choosing points $\{\Phi(e_1), ..., \Phi(e_N)\}$ independently uniformly at random from I, and taking C to be the induced order: $e_i \prec e_j$ in C if $\Phi(e_i) < \Phi(e_j)$ in the causal order on the manifold. Let (u_i, v_i) denote the coordinates of the sprinkled element $\Phi(e_i)$. With probability 1, all the values u_i and v_i are different. As described above, C is the intersection of the linear orders U, V obtained from these u and v values. Each of the pairs (u_i, v_i) is chosen uniformly over the rectangle I, so the coordinates u_i and v_i are independent of each other, and of all other choices. Thus no permutation of the elements of C can be more likely to occur as the order U than any other, i.e., the random linear order U is distributed uniformly over all linear orders of elements of C. The order V is also uniform over the set of all linear orders, and is independent of U. The process of taking a sprinkling and deriving a (labelled) causet from it is therefore equivalent to taking a random causet according to $P(N)$.

This means that a "typical" random order from $P(N)$ corresponds (in the sense of a faithful embedding) to an interval of 2D Minkowski space of volume NV_c. For a spacetime with non-trivial conformal factor, while the process of sprinkling is still a random process, there will in general be correlations in the u and v values. Hence, sprinklings into such spacetimes which differ from flat spacetime at scales much larger than the cut-off, are not equivalent to the random 2D orders $P(N)$.

The following result was first proved by El-Zahar and Sauer [23], and was stated in this form by Winkler [24], who gave an alternative proof and considered the (more complicated) labelled case as well.

Theorem 1 *Let Φ be an isomorphism-invariant statement about 2D orders which has a limiting probability either in $P(N)$ or in $U(N)$. Then a limiting probability exists in the other case as well and the two probabilities are equal.*

The proofs of El-Zahar and Sauer, and of Winkler, also give that the number of N-element 2D orders is $N!/2(1 + o(1))$, and that almost all of them have a unique representation, up to isomorphism, as an intersection of two linear orders. Here, "limiting probability" refers to the probability in the $N \to \infty$ limit. From our discussion above, it then follows that as $N \to \infty$, the partition function (7) is dominated by causets which faithfully embed into an interval of Minkowski spacetime of volume $V = NV_c$. This emergence of manifoldlike causets in an apparently featureless partition function is surprising, to say the least. Dominance of a class of

configurations in the partition function has a standard interpretation in quantum theory, which translates in our case to the statement that *2D Minkowski spacetime is a prediction of our theory.*

The large N limit taken above can be interpreted as a large volume, if the discreteness scale V_c is held constant, or a continuum limit if, instead, the total volume V is held constant. We see in the above model that, in the continuum approximation, fluctuations die out altogether, with flat space dominating. Thus, despite the possibility of having no classical limit at all in 2D, the continuum approximation is actually a classical limit. This is not a feature to be found in other 2D quantum gravity models [12]. However, it *is* something that is desirable in a model of 4D CSQG, since the discreteness scale is of order the quantum gravity scale. These results are therefore interesting from this point of view. The size of quantum fluctuations at given N remains to be calculated and will require numerical analysis. We leave this for future investigations.

A similar model may be constructed for the cylinder topology $S^1 \times \mathbb{R}$, the other class of 2 dimensional spacetimes with fixed topology. Causets on the cylinder can be partly characterised by the existence of the crown sub-posets described above, resulting in a non-vanishing first Betti number (which means that they are not 2D orders). However, we know of no definitive characterisation of such "cylinder" posets analogous to the 2D orders discussed above for the disc topology. It would be of great interest to check if a continuum spacetime is also emergent for this class of causets. Such a model would help in a more straightforward comparison with existing 2D quantum gravity models – would the radius of the cylinder fluctuate in the continuum limit as in other models, or would a classical limit be obtained?

In this model, as in other lower-dimensional models, many of the problems that exist in the 4D case are avoided, but not always in a way that immediately suggests answers to the 4D problems. Nonetheless, there are some lessons from 2D to be learned. It is an encouraging and non-trivial fact that, in the set of 2D orders, sprinklings of flat space naturally dominate. Once the restriction to 2D orders is made, non-manifoldlike causets are no longer entropically preferred. In 4D, causets that can be embedded into intervals of Minkowski space are known as "4D sphere orders" [18]. It would be of great interest to know whether any analog of the El-Zahar/Sauer result holds here: we can define the probability spaces $U(N)$ and $P(N)$ as in the 2D case, where $P(N)$ now refers to sprinkling into an interval of 4D

Minkowski space, and ask how these are related. It is probably too much to expect that every statement about 4D sphere orders has the same limiting probability in the two models, but nevertheless it may well be true that a causet drawn from $U(N)$ typically corresponds to an interval in the manifold, in the sense considered here. If so, this bodes well for the entropy problem in CSQG.

References

[1] J. Ambjorn, J. Jurkiewicz and R. Loll, "Quantum gravity, or the art of building spacetime," in D. Oriti (ed), "*Approaches to Quantum Gravity - Toward a new understanding of space and time*", Cambridge University Press, 2006; L. Smolin, "The case for background independence,", in D. Rickles et al. (ed.), *" The structural foundations of quantum gravity"*, Oxford University Press, USA (2007), 196-239. ; F. Dowker, "Causal sets and the deep structure of spacetime," in A. Ashtekar (ed), *"100 Years of Relativity. Space-time Structure: Einstein and Beyond"*, World Scientific, (2005).

[2] L. Bombelli, J. Henson and R. D. Sorkin, arXiv:gr-qc/0605006.

[3] R. D. Sorkin, Int. J. Theor. Phys. **36**, 2759 (1997); M. Ahmed, S. Dodelson, P. B. Greene and R. Sorkin, Phys. Rev. D **69**, 103523 (2004).

[4] L. Bombelli, J. Lee, D. Meyer and R. D. Sorkin, Phys. Rev. Lett. **59**, 521 (1987).

[5] L. Bombelli. Talk at "Causet 2006: A Topical School funded by the European Network on Random Geometry", Imperial College, London, U.K., September 18-22, 2006.

[6] D. P. Rideout and R. D. Sorkin, Phys. Rev. D **61**, 024002 (2000).

[7] J. W. Barrett and L. Crane, Class. Quant. Grav. **17**, 3101 (2000) ; J. Ambjorn, J. Correia, C. Kristjansen and R. Loll, Phys. Lett. B **475**, 24 (2000).

[8] T. Regge, Nuovo Cimento, **19**, 558, (1961); R. D. Sorkin, Phys. Rev. D **12**, 385 (1975).

[9] J. Henson, Stud. Hist. Philos. Mod. Phys. **36**, 519 (2005).

[10] R. D. Sorkin, "Does locality fail at intermediate length scales?", in D. Oriti (ed), "*Approaches to Quantum Gravity - Toward a new understanding of space and time*", Cambridge University Press, 2006; J. Henson, "The causal set approach to quantum gravity," in D. Oriti (ed), *"Approaches to Quantum Gravity - Toward a new understanding of space and time"*, Cambridge University Press, 2006.

[11] F. Markopoulou and L. Smolin, Nucl. Phys. B **508**, 409 (1997).

[12] F. David, arXiv:hep-th/9303127; F. David (Saclay), Lectures at 8th Jerusalem Winter School for Theoretical Physics, Two-Dimensional Gravity and Random Surfaces, Jerusalem, Israel, Dec 27 - Jan 4, 1991. Published in Jerusalem Gravity 1990:0125-141 (QC178:J4:1990).

[13] W. G. Unruh, Phys. Rev. D **40**, 1048 (1989). R. D. Sorkin, Int. J. Theor. Phys. **33**, 523 (1994).

[14] G. Hayward, Phys. Rev. D **47**, 3275, (1993).

[15] J. B. Hartle and R. Sorkin, Gen. Rel. Grav. **13**, 541 (1981).

[16] G. S. Birman and K. Nomizu, Mich. Math. J. **31**, 77-81, (1984).

[17] P. R. Law, Rocky Mountain Journal of Mathematics, **22**, 1365, (1992).

[18] William T. Trotter, "Combinatorics and Partially Ordered Sets", The Johns Hopkins University Press, 1992.

[19] S. Major, D. Rideout, S. Surya, Journal of Math. Phys. **48**, 032501 (2007).

[20] D. Kleitman and B. L. Rothschild, Trans. Am. Math. Soc. **205**, 205 (1975).

[21] Peter Winkler, Order **1**, 317, (1985).

[22] D. Stoyan, W.S. Kendall, J. Mecke, "Stochastic Geometry and Its Applications", John Wiley & Sons, 1996.

[23] M.H. El-Zahar and N.W. Sauer, Order **5**, 239, (1988).

[24] P. Winkler, Order **7**, 329, (1991).